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Density Segregation in Vibrated Granular Beds with Bumpy Surfaces

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Segregation of granular materials by virtue of density or size is a commonly encountered phenomenon in nature. Despite its widespread interest among many researchers in recent years, a complete and unified understanding of granular segregation remains elusive to date. Using molecular dynamics simulations, we report a novel technique of inducing density segregation in a binary mixture of granular materials subjected to vibrations by the use of a bumpy vibrating base. Density segregation in the vertical directions may be induced by oscillating the bumpy base composed of discrete solid particles vertically or horizontally. In both cases, lighter particles tended to rise to the top of the granular bed and form a layer above the heavier particles. We suggest that differences in granular temperature profiles arising from the two different modes of vibrations may play an important role in determining the extent of density segregation occurring in binary granular mixtures. *AIChE J*, 56: 2588–2597, 2010
Keywords: vibrated bed, granular mixtures, density segregation, discrete element method, numerical simulation

Introduction

A bed of granular materials subjected to vertical vibrations exhibits a variety of interesting phenomena and has been the focus of many research studies in recent years. It is well established that granular mixtures are capable of undergoing segregation by virtue of a variety of physical property differences. These include segregations induced by particle size,[1–7] density,[2,8–12] and inelasticity or differences in coefficient of restitution.[13] The Brazil nut effect and reversed Brazil nut effect, whereby a larger intruder in a granular bed of smaller particles either rises or sinks under the effect of external vibrations, have also been the subject of considerable interest among research workers in this area.[14–19] In general, these segregation phenomena have been attributed to various physical factors such as friction and shaking properties,[2,20] competition between percolation and condensation,[3] "void-filling" mechanism,[4] combination of void-filling, sidewall-driven convection rolls, and thermal diffusion,[5] competition between buoyancy and sidewall-driven convection,[6] effect of interstitial air,[8,10,12,16] combination of inertia, convection, and buoyancy,[9] side-wall convection effects,[14] and inertia assisted by Reynolds dilatancy.[15]

Despite the fact that various aspects of the vibrated granular bed system pertaining to granular segregation have been investigated fairly extensively in the research literature, others in contrast have remained conspicuously unexplored so far. One such aspect relates to the nature of the base on which granular materials are oscillated. It has been reported recently that the bulk behaviors of granular materials subjected to vertical vibrations on a flat or bumpy base are qualitatively and quantitatively similar.[21] In this study, we explore the effect of applying a bumpy vibrating base on the segregation behaviors of binary granular mixtures using molecular dynamics simulations. Both vertical and horizontal vibrations of the bumpy base will be studied, and we show

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that granular temperature profiles developed in the granular beds during vibrations may also be an important parameter determining the extent of segregation of the binary granular mixtures. In the following section, the numerical method applied and physical system of interest to this study will be described.

Model

The molecular dynamics approach to modeling of granular systems, otherwise known as the discrete element method (DEM), has been applied extensively for studies of various aspects of granular behavior. The methodology of DEM and its corresponding governing equations have also been presented numerous times in the research literature,[21–23] and only a brief description will be presented here for sake of completeness. For a comprehensive review, the interested reader is referred to a recent review article by Zhu et al.[24]

The translational and rotational motions of individual solid particles are governed by Newton's laws of motion:

$$m_i \frac{dv_i}{dt} = \sum_{j=1}^{N} (f_{c,ij} + f_{d,ij}) + m_i g, \tag{1}$$

$$I_i \frac{d\omega_i}{dt} = \sum_{j=1}^{N} T_{ij}, \tag{2}$$

where m_i and v_i are the mass and velocity of the ith particle, respectively, N is the number of particles in contact with the ith particle, $f_{c,ij}$ and $f_{d,ij}$ are the contact and viscous contact damping forces, respectively, I_i is the moment of inertia of the ith particle, ω_i is its angular velocity, and T_{ij} is the torque arising from contact forces, which causes the particle to rotate.

Contact and damping forces have to be calculated using force-displacement models that relate such forces to the relative positions, velocities, and angular velocities of the colliding particles. Following Lim,[21,22] a linear spring-and-dashpot model is implemented for the calculation of these collision forces. With such a closure, interparticle collisions are modeled as compressions of a perfectly elastic spring, while the inelasticities associated with such collisions are modeled by the damping of energy in the dashpot component of the model. Collisions between particles and a wall may be handled in a similar manner but with the latter not incurring any change in its momentum. In other words, a wall at the point of contact with a particle may be treated as another particle but with an infinite amount of inertia. The normal ($f_{cn,ij}$, $f_{dn,ij}$) and tangential ($f_{ct,ij}$, $f_{dt,ij}$) components of the contact and damping forces are calculated according to the following equations:

$$f_{cn,ij} = -(\kappa_{n,i}\delta_{n,ij})n_i \tag{3}$$

$$f_{ct,ij} = -(\kappa_{t,i}\delta_{t,ij})t_i \tag{4}$$

$$f_{dn,ij} = -\eta_{n,i}(v_r \cdot n_i)n_i \tag{5}$$

$$f_{dt,ij} = -\eta_{t,i}\{(v_r \cdot t_i)t_i + (\omega_i \times R_i - \omega_j \times R_j)\}, \tag{6}$$

where $\kappa_{n,i}$, $\delta_{n,ij}$, n_i, and $\eta_{n,i}$ and $\kappa_{t,i}$, $\delta_{t,ij}$, t_i, and $\eta_{t,i}$ are the spring constants, displacements between particles, unit vectors, and viscous contact damping coefficients in the normal and tangential directions, respectively, v_r is the relative velocity between particles, and R_i and R_j are the radii of particles i and j, respectively. If $|f_{ct,ij}| > |f_{cn,ij}| \tan\phi$, then "slippage" between two contacting surfaces is simulated based on Coulomb-type friction law, i.e., $|f_{ct,ij}| = |f_{cn,ij}| \tan\phi$, where $\tan\phi$ is analogous to the coefficient of friction.

Table 1. Material Properties and System Parameters

Shape of particles	Spherical
Number of particles	1000
Particle diameter	0.5 mm
Particle density	1.31, 2.87, 4.45, 8.37, and 18.0 g cm^{-3}
Coefficient of restitution, ε	0.9
Coefficient of friction	0.3
System dimensions	35 mm width × 140 mm height
Vibrating frequency, f	90 Hz
Vibrating amplitude, A	0.184, 0.368, 0.736, 1.104, 1.472, and 1.84 mm
Dimensionless acceleration, Γ	6.0, 12, 24, 36, 48, and 60
Simulation time step	10^{-8} s

The vibrated granular bed systems of interest to this study are the four types of binary mixtures studied experimentally by Shi et al.[11] recently. We consider a pseudo-three-dimensional equivalent of their system (35 mm width × 140 mm height) of one particle thickness in the spanwise direction. Two vertical walls are imposed on each end of the system in the horizontal direction so as to correspond as closely as possible to the actual physical system used by the previous researchers. Under random packing conditions and with the entire system at rest, this pseudo-three-dimensional volume is estimated to contain a reasonable 1000 particles each of diameter 0.5 mm, amounting to about 14–16 layers of granular materials. The base of the system is simulated to undergo a sinusoidal motion such that its displacement from the equilibrium position is described by $A\sin(\omega t)$, where A is the amplitude of the oscillatory motion, $\omega = 2\pi f$ is the angular frequency, and t is time. The value of coefficient of restitution for both particle–particle and particle–base collisions is $\varepsilon = 0.9$. The vibrating frequency and amplitude are $f = 90$ Hz and $A = 0.184$ mm, respectively, giving the value of the dimensionless acceleration, $\Gamma = \frac{4\pi^2 f^2 A}{g} = 6.0$. Table 1 provides a summary of the pertinent material properties and simulation parameters used. As will be explained later, the entire set of simulation was then repeated with the vibrating base replaced by particles undergoing a similar type of imposed oscillatory motion to represent a "bumpy" vibrating base.

Results and Discussion

We first show in Figure 1 a simulation of the physical system studied experimentally by Shi et al.[11] Blue and red particles represent lighter and heavier particles, respectively, with the former made of aluminum oxide of density 1.31 g cm^{-3}. The heavier particles in the four panels are made of zirconium oxide (2.87 g cm^{-3}), titanium alloy (4.45 g cm^{-3}), cobalt-chromium-molybdenum alloy (8.37 g cm^{-3}), and tungsten alloy (18.0 g cm^{-3}). A good agreement is observed between the present simulations and experiments of the previous research workers with regards to the states of

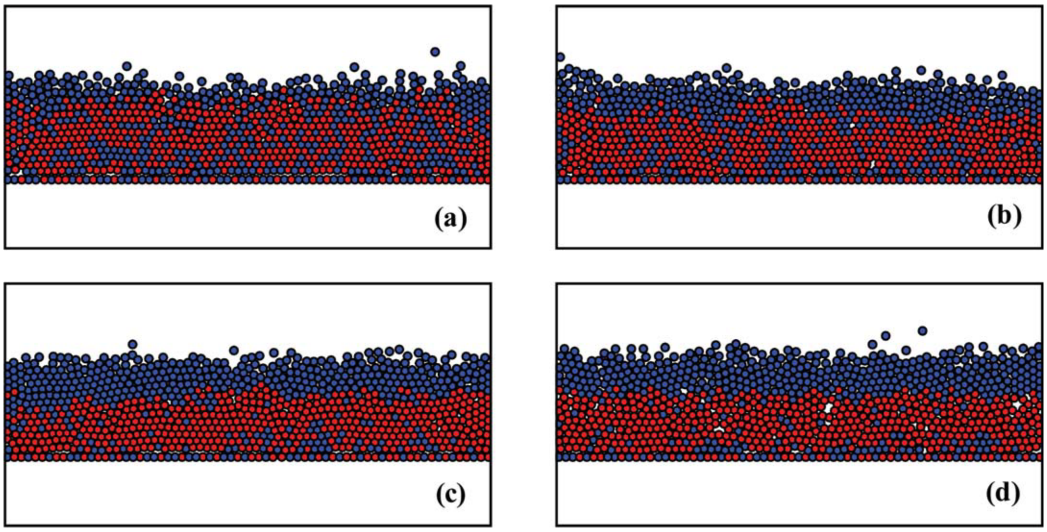

Figure 1. Molecular dynamics simulations of the segregated states of binary granular mixtures.

The lighter particles (blue) are made of aluminum oxide (1.31 g cm^{-3}) and the heavier ones (red) are made of (a) zirconium oxide (2.87 g cm^{-3}), (b) titanium alloy (4.45 g cm^{-3}), (c) cobalt-chromium-molybdenum alloy (8.37 g cm^{-3}), and (d) tungsten alloy (18.0 g cm^{-3}). $f = 90$ Hz, $A = 0.184$ mm, and $\Gamma = 6.0$. A good agreement with the experimental observations of Shi et al.[11] is obtained.

segregation of the various binary mixtures of granular materials (see Figure 1 of Ref. [11]). There is a tendency for lighter particles to rise through the granular bed to form a layer above the heavier particles. The extent of such density segregation increases with increasing difference in density between light and heavy particles. In a recently completed study, it was shown that a vibrated granular bed system with a flat base is qualitatively and quantitatively equivalent to one with a bumpy base composed of discrete solid particles undergoing the same type of imposed, oscillatory motion.[21] Here, we show that this interesting property of vibrated granular beds is also applicable to the phenomenon of density segregation. Figure 2 shows the result of vibrating the same four types of binary mixtures of granular materials on a bumpy base composed of 5-mm solid particles. The same sinusoidal motion is imposed on this row of particles to simulate a vibrating base with a bumpy surface. The same states of segregation may be observed, with the uneven nature of the vibrating base seeming to exert a negligible effect on the bulk behavior of the granular mixtures. A more quantitative characterization of the states of segregation may be inferred from Figure 2e, which shows the number fraction distributions of light particles along the vertical direction of the bed. It may be observed that the larger the difference in density between light and heavy particles, the more marked the extent of segregation. In particular, the bed containing zirconium oxide as heavy particles exhibits a smooth increase in number fraction of light particles as one proceeds from the bottom to the top of the bed. On the other hand, the bed containing tungsten alloy shows a sharp rise in number fraction over a small distance near the central region of the bed.

More interestingly, we show next the result of changing the mode of vibration of the bumpy base to that of horizontal vibration. The same 5-mm solid particles composing the base are oscillated in the horizontal direction at the same frequency and amplitude while maintaining all other conditions in the system identical to the previous cases. Figure 3 shows that density segregations only occur minimally in the third and fourth types of binary granular mixtures. The high tendency of cobalt-chromium-molybdenum alloy–aluminum oxide mixture and titanium alloy–aluminum oxide mixture to undergo vertical segregation is manifest in the thin layer of light particles formed at the surface of the respective granular beds. Plots of the number fraction distributions of light particles in the vertical direction for the four types of binary mixtures (Figure 3e) confirm quantitatively that the granular beds are composed largely of random mixtures. In recent experimental[25,26] and computational studies,[27,28] it has been demonstrated that granular beds subjected to vibrations at specific amplitudes and frequencies may undergo a process referred to as densification rather than fluidization. The solid particles assemble into crystalline structures, which hinder relative motions within the granular bed. Figure 3 shows that this is likely to be the mechanism responsible for the limited extents of segregation observed in all four types of binary granular mixtures subjected to horizontal vibrations on a bumpy base at $\Gamma = 6.0$. In contrast, when the vibrating conditions applied are changed to $f = 90$ Hz, $A = 1.84$ mm, and $\Gamma = 60$, all four types of binary mixtures undergo significant density segregations (Figure 4). Because of the vigorous shaking conditions imposed, the granular beds are partially fluidized forming a region of lower solid concentration near the horizontally vibrating base where particles are highly energetic. This is in sharp contrast to the previous case where solid particles at the bottom of the bed remain more or less adhered to the vibrating base during operation, whereas others throughout the bed are induced to vibrate about their mean positions only (Figure 3). These observations seem to indicate that the energy imparted by an oscillating base to a binary mixture of particles in a vibrated granular bed system may play an important role in inducing segregation behaviors. As such, a quantitative analysis of the granular temperature profiles of such systems may provide the key toward understanding the density segregation

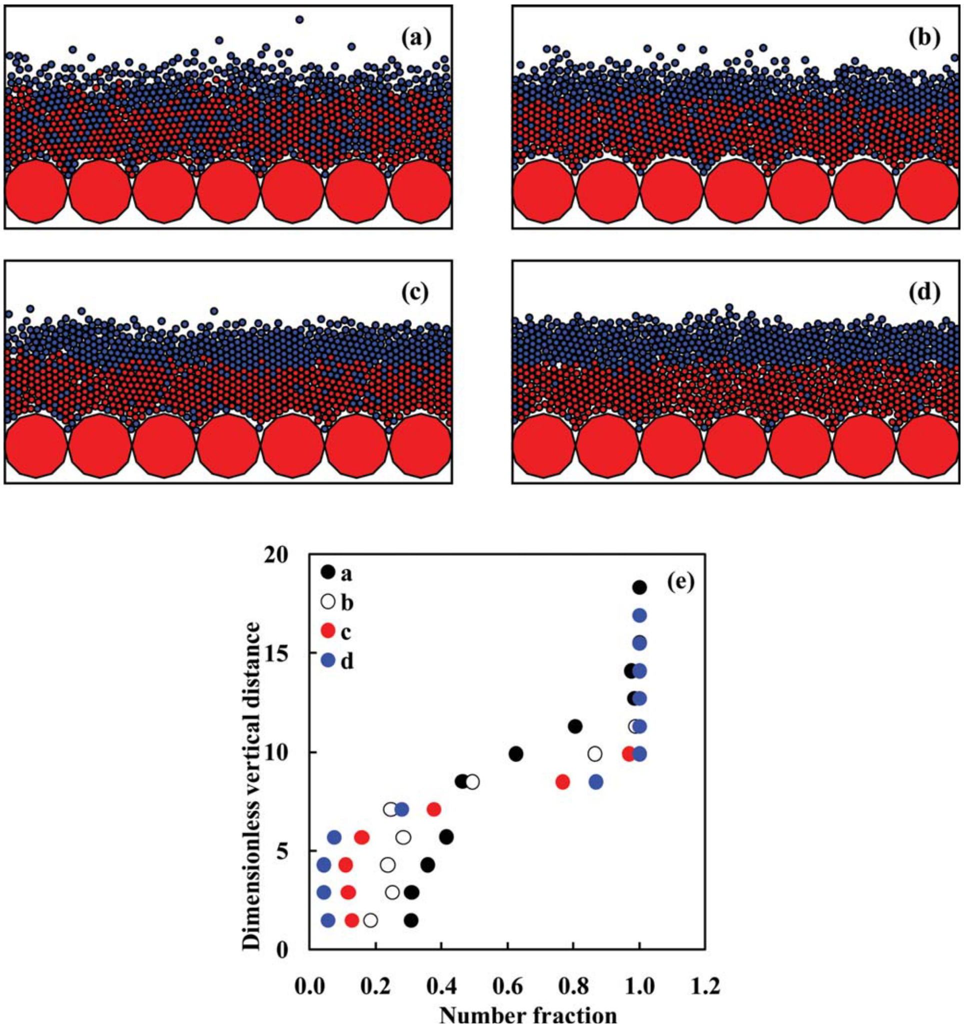

Figure 2. Segregated states of the four types of binary granular mixtures (a) zirconium oxide, (b) titanium alloy, (c) cobalt-chromium-molybdenum alloy, and (d) tungsten alloy subjected to vertical vibrations on a bumpy base composed of 5-mm solid particles.

The same type of density segregation where lighter particles rise through the granular bed to form a layer above the heavier particles is observed. (e) Number fraction distributions of light particles along the vertical direction of the bed for the four respective types of granular mixtures. Vertical distances have been nondimensionalized by particle diameter.

behaviors observed in this study, and this will be discussed in the next section.

The two types of oscillatory motions (vertical or horizontal) of the bumpy base as well as the operating conditions imposed will result in significant differences in granular temperature profiles of the vibrated bed. Figures 5a, b show that under vertical oscillations of the bumpy base, the two components of granular temperatures T_y and T_x increase gradually with vertical distance along the bed. Here, $T_y = \frac{1}{2n}\sum_{i=1}^{n}\left(v_{y,i} - \overline{v_y}\right)^2$ and $T_x = \frac{1}{2n}\sum_{i=1}^{n}\left(v_{x,i} - \overline{v_x}\right)^2$, where $v_{y,i}$ and $v_{x,i}$ are the vertical and horizontal components of velocity of the ith particle, respectively, $\overline{\boldsymbol{v_y}}$ and $\overline{\boldsymbol{v_x}}$ are the corresponding mean velocities. Generally, granular temperatures are fairly isotropic for all four types of binary mixtures when subjected to vertical vibrations of the bumpy base, as can be seen from the similar profiles of T_y and T_x. Both components range from about 0 to 1.5×10^{-3} m^2 s^{-2} over a vertical distance of about 20 particle diameters. On the other hand, Figures 5c, d show that T_y and T_x profiles for the four binary mixtures subjected to horizontal vibrations of the bumpy base exhibit a maximum point at a vertical position of about 5 particle diameters. Beyond this point, both T_y and T_x decrease smoothly with increasing vertical distance, which is in marked contrast to the previous profiles. Furthermore, granular temperatures are slightly less isotropic in this case as the maximum value of T_y is observed to be about 2.0×10^{-3} m^2 s^{-2}, while that of T_x is about 1.0×10^{-3} m^2 s^{-2}. These significant differences in granular temperature profiles may be indicative of mechanistic differences underlying the behaviors of the granular beds when subjected to the two modes of vibrations. As such, we suggest that granular temperature may be a key property of a vibrated granular mixture, which governs its segregation behavior. In contrast with a conventional vibrated granular bed system with a flat vibrating base, it seems that one with a bumpy base is more likely to generate anisotropy in granular temperature profiles

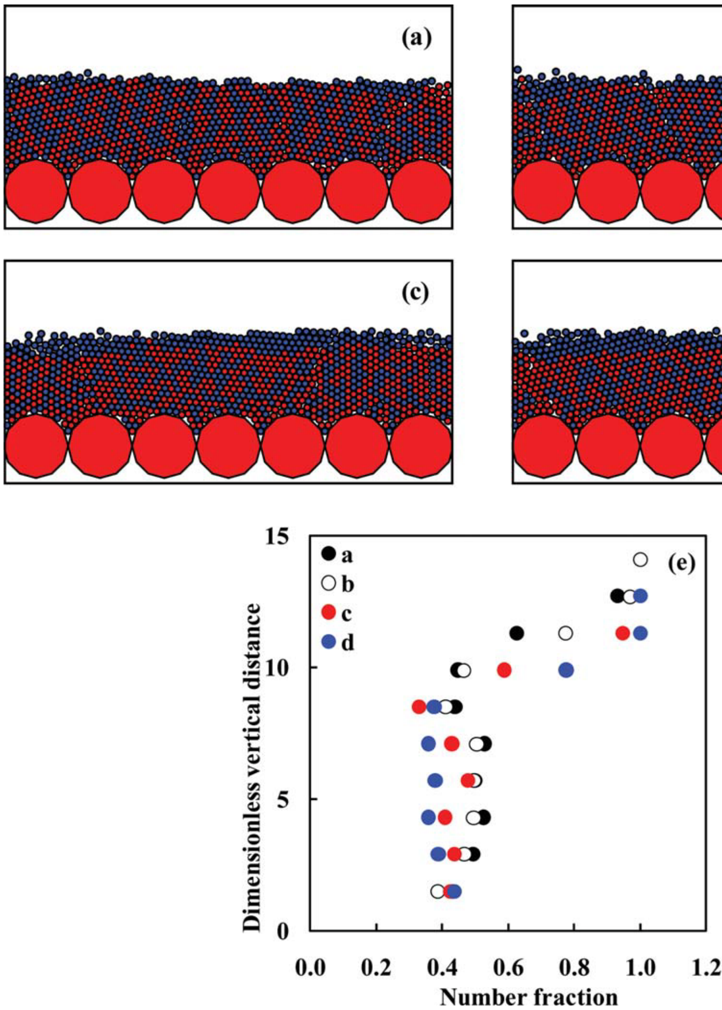

Figure 3. Segregated states of the four types of binary granular mixtures (a) zirconium oxide, (b) titanium alloy, (c) cobalt-chromium-molybdenum alloy, and (d) tungsten alloy subjected to horizontal vibrations on a bumpy base at the same vibrating frequency and amplitude as the previous case.

In contrast with the previous case, density segregation in the vertical direction occurs minimally. (e) Number fraction distributions of light particles along the vertical direction of the bed for the four respective types of granular mixtures indicate that the granular beds are composed largely of random mixtures. Vertical distances have been nondimensionalized by particle diameter.

throughout the bed. This is due to the possibility of oblique impacts between solid particles in the bed and the uneven, vibrating surface. Although the relationship between bumpiness of the base and extent of anisotropy in granular temperatures is not completely understood yet in the research literature, it may be imagined that both components of granular temperature generated by the base are important in determining whether the entire granular bed undergoes densification or fluidization. This would in turn determine the amount of relative particle motion that can occur within the bed and ultimately the extent of density segregation of a binary granular mixture. The granular temperature profiles for all four binary mixtures under the more vigorous horizontal vibration condition of $\Gamma = 60$ increase to approximately an order of magnitude higher than in the corresponding previous cases as shown in Figures 5e, f. Values of both T_y and T_x reach as high as 30×10^{-3} m^2 s^{-2} near the vibrating base and decrease smoothly with increasing vertical distance. Interestingly, granular temperatures revert to being more isotropic at such operating conditions. In comparing the various panels of Figure 5, we observe that granular beds subjected to vertical vibrations exhibit positive granular temperature gradients with particles near the surface having the highest temperatures, whereas beds subjected to horizontal vibrations by a bumpy base are characterized by negative granular temperature gradients with particles near the base having the highest temperatures instead. Second, in horizontally vibrated granular beds on bumpy surfaces, higher granular temperatures seem to be more effective at inducing density segregations in binary mixtures. To our knowledge, these relationships between granular temperatures and density segregations that have been observed through the use of a bumpy vibrating base in this study do not seem to have been reported in previous experimental or numerical studies.

With these observations, it was deemed prudent to then investigate the effects of the dimensionless acceleration parameter Γ on the density segregation behaviors of the four types of binary granular mixtures. To this end, the bumpy

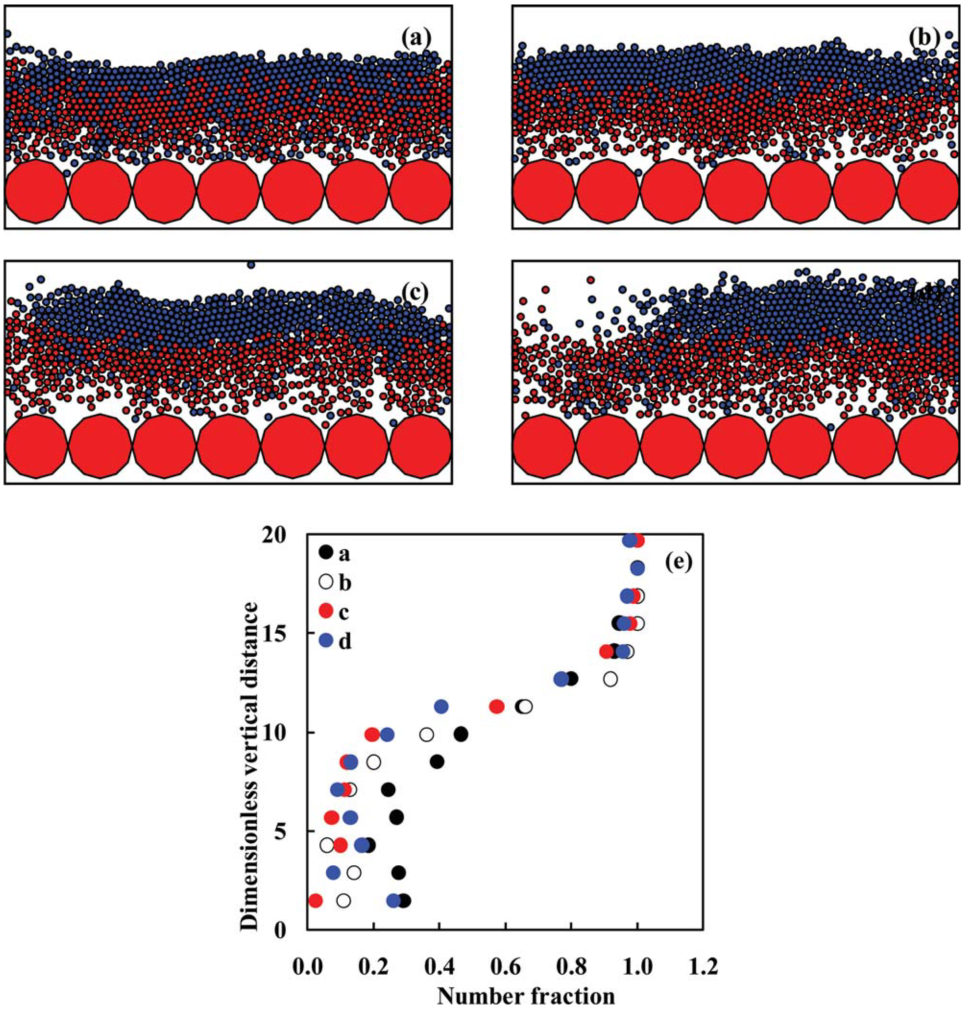

Figure 4. Segregated states of the four types of binary granular mixtures (a) zirconium oxide, (b) titanium alloy, (c) cobalt-chromium-molybdenum alloy, and (d) tungsten alloy subjected to horizontal vibrations on a bumpy base at f = 90 Hz, A = 1.84 mm, and Γ = 60.

The beds are partially fluidized during operation because of the more vigorous vibrating conditions. (e) Number fraction distributions of light particles along the vertical direction of the bed for the four respective types of granular mixtures. Vertical distances have been nondimensionalized by particle diameter.

base simulations with horizontal vibrations were repeated for Γ = 12, 24, 36, and 48 by varying the vibrating amplitude while keeping the frequency and all other parameters constant. It may be noted that cases for Γ = 6.0 and 60 have been presented in Figures 3e and 4e, respectively. Figures 6a, b show the number fraction distributions of light particles along the vertical direction achieved corresponding to the vibrating conditions Γ = 12 and 36, respectively. In each panel, the four symbols denote the four types of binary granular mixtures. As with previous cases, it may be seen that granular beds with heavier particles made of cobalt-chromium-molybdenum alloy and tungsten alloy undergo larger extents of segregation under all vibrating conditions because of the larger density differences between light and heavy particles. In addition, the extents of segregation for all four binary mixtures also increase with increasing values of Γ such that at Γ = 48 (data not shown for brevity), the distinction between large and small density difference between light and heavy particles has become diminished. Figures 7a, b show profiles of vertical component of granular temperatures along the vertical direction corresponding to the same two vibrating conditions, respectively, and Figures 8a, b show the corresponding horizontal component of granular temperatures. In general, granular temperature profiles are fairly isotropic for all four vibrating conditions. The characteristic negative temperature gradient in the vertical direction with highest temperatures occurring near the vibrating base and lowest temperatures occurring at the surface of the bed is also apparent from both Figures 7 and 8. More interestingly, despite similarities in terms of shapes of the granular temperature curves, maximum values of granular temperatures show an increase from about 2.0×10^{-3}, 8.0×10^{-3}, 25×10^{-3} to about 40×10^{-3} m^2 s^{-2} as Γ increases from 12, 24, 36 to 48, respectively. This corresponds directly with the increase in extents of density segregation observed earlier in Figure 6 and supports the previous claim that higher granular temperatures in horizontally vibrated granular beds on bumpy surfaces are associated with larger

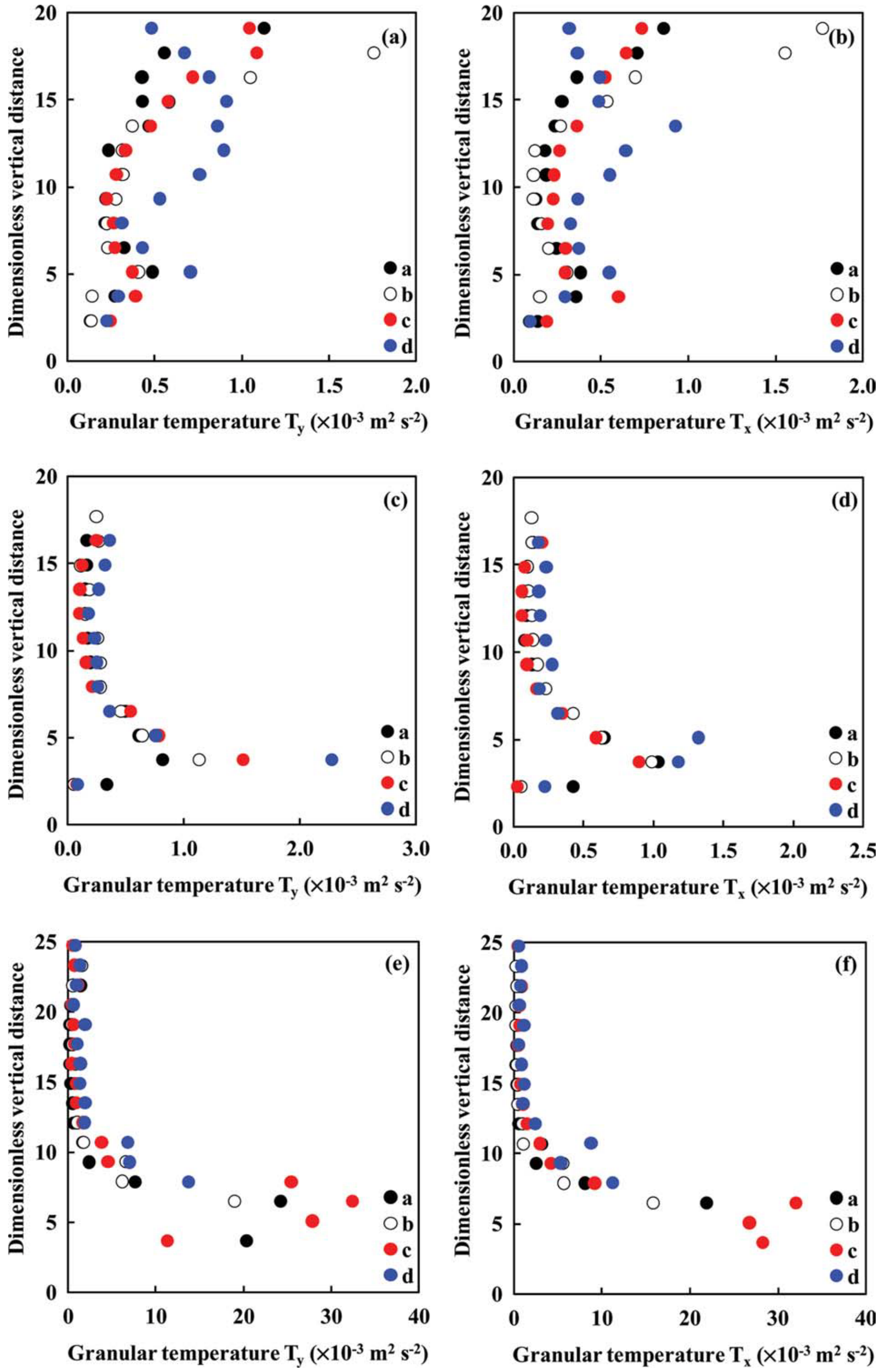

Figure 5. Spatially averaged granular temperature profiles (a) T_y and (b) T_x for the four types of binary granular mixtures subjected to vertical vibrations on a bumpy base at f = 90 Hz, A = 0.184 mm, and Γ = 6.0.

Granular temperatures are fairly isotropic for all four types of binary mixtures when subjected to vertical oscillations, and both components T_y and T_x increase gradually with vertical distance along the bed. In contrast, the corresponding (c) T_y and (d) T_x profiles for the four binary mixtures subjected to horizontal vibrations of the bumpy base at the same vibrating frequency and amplitude exhibit a maximum point beyond which both T_y and T_x decrease smoothly with increasing vertical distance. Granular temperatures are also less isotropic in this case. With horizontal vibrations at $f = 90$ Hz, $A = 1.84$ mm, and $\Gamma = 60$, both (e) T_y and (f) T_x profiles for the four binary mixtures are approximately an order of magnitude higher than in the corresponding previous cases. Granular temperatures revert to being fairly isotropic at such operating conditions. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

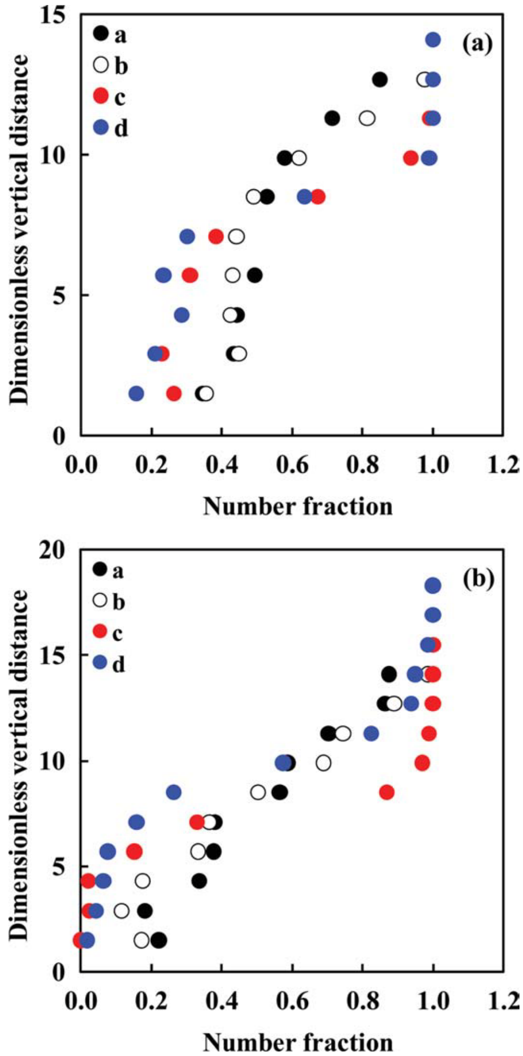

Figure 6. Number fraction distributions of light particles along the vertical direction of the bed for the four respective types of granular mixtures.

The two panels show the result of subjecting the granular mixtures to horizontal vibration on a bumpy base at $f = 90$ Hz, (a) $A = 0.368$ mm ($\Gamma = 12$) and (b) $A = 1.104$ mm ($\Gamma = 36$). Symbols in each panel denote the different types of granular mixtures as considered in all previous cases. Vertical distances have been nondimensionalized by particle diameter. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

extents of density segregation. A mechanistic understanding of this observation may be obtained by reference to an earlier study[29] in which it was reported that both higher granular temperatures and diffusion coefficients of solid particles in a vibrated granular bed are associated with more vigorous vibration conditions as defined by the dimensionless acceleration parameter. At higher values of Γ, velocity fluctuations of solid particles are larger because of larger energy input from the vibrating base, and there are higher tendencies for such energetic particles to move away from their original positions within the granular bed. In a binary granular mixture, such a mechanism is expected to enhance the natural tendencies for higher (or lower) density particles to descend (or ascend) through the bed, thus leading to density segregation.

Conclusions

We have investigated the density segregation behaviors of various binary granular mixtures subjected to vertical and horizontal vibrations on a bumpy vibrating surface using molecular dynamics simulations. The binary mixtures consisted of light and heavy particles with varying density differences. When subjected to vertical vibrations on a flat or bumpy surface, lighter particles tended to rise through the granular bed

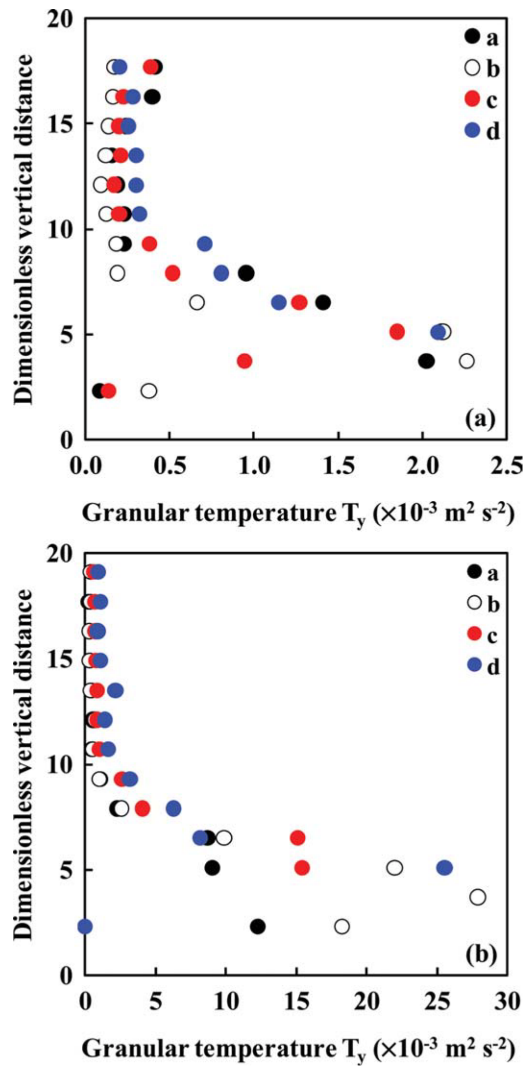

Figure 7. Profiles of spatially averaged vertical components of granular temperature T_y for the four types of binary granular mixtures subjected to horizontal vibrations on a bumpy base at $f = 90$ Hz, (a) $A = 0.368$ mm ($\Gamma = 12$) and (b) $A = 1.104$ mm ($\Gamma = 36$).

Symbols in each panel denote the different types of granular mixtures as considered in all previous cases. Vertical distances have been nondimensionalized by particle diameter. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

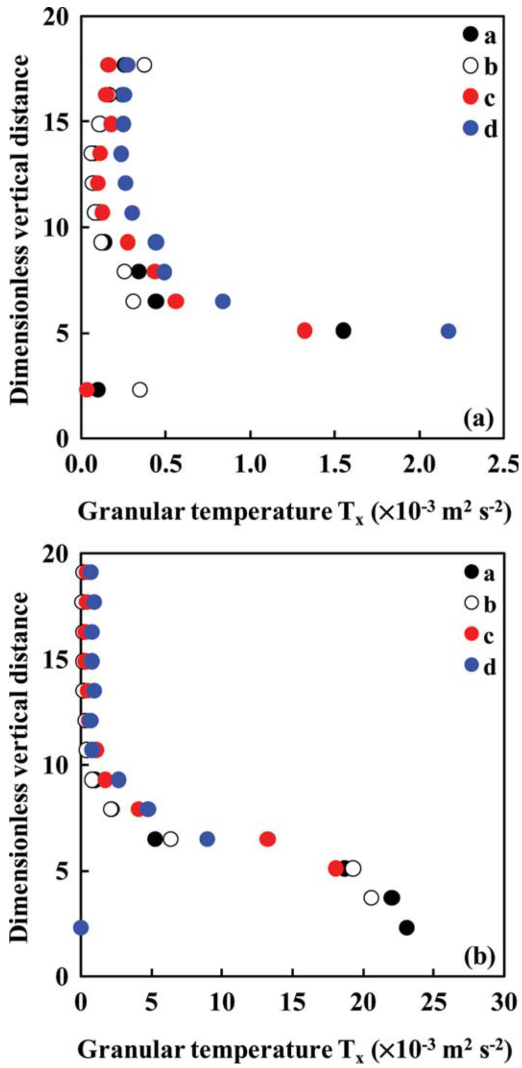

Figure 8. Profiles of spatially averaged horizontal components of granular temperature T_x for the four types of binary granular mixtures subjected to horizontal vibrations on a bumpy base at f = 90 Hz, (a) A = 0.368 mm (Γ = 12) and (b) A = 1.104 mm (Γ = 36).

Symbols in each panel denote the different types of granular mixtures as considered in all previous cases. Vertical distances have been nondimensionalized by particle diameter. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

to form a layer above the heavier particles. The extents of segregation observed increased with increasing density differences between the two types of particles with no apparent disparity in segregation behaviors arising from the bumpiness of a bumpy vibrating surface compared with a flat surface. The same type of segregation behavior could be induced through horizontal vibrations of the bumpy surface. It was observed that the extent of density segregation increased with increasing intensity of vibration characterized by the magnitude of the dimensionless acceleration parameter. This is also associated with a corresponding increase in magnitudes of granular temperatures within the vibrated beds. In comparing between the two modes of vibration, we observed that granular beds subjected to vertical vibrations exhibited positive granular temperature gradients with particles near the surface having the highest temperatures, whereas beds subjected to horizontal vibrations by a bumpy base were characterized by negative granular temperature gradients with particles near the base having the highest temperatures instead. Furthermore, higher granular temperatures seemed to be more effective at inducing density segregations in binary mixtures subjected to horizontal vibrations on bumpy surfaces. We suggest that granular temperature may be a key property of a vibrated granular mixture, which governs its segregation behavior. Finally, this study has also demonstrated the possibility of a new method of injecting energy into granular mixtures through the use of horizontally vibrated bumpy surfaces so as to induce density segregations in those mixtures. Further considerations of such segregation behaviors in various closely related systems such as flows of granular mixtures down inclined planes with bumpy surfaces or in rotating tumblers with bumpy walls may constitute possible subjects of future research studies.

Acknowledgments

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